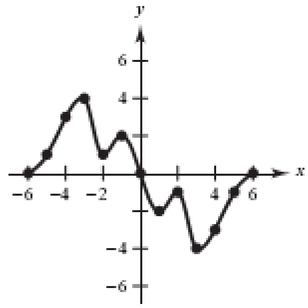
Answer Key for Today's 11-9-07 Homework Assignment - Chapter 3 Review Pages 242-244

[3.99 Review] p.242-244 #2,8,16,18,24,26,28,36,38,42,52,58,72,87

(c)



2. (a) $f(4) = -f(-4) = -3$

At least six critical numbers on $(-6, 6)$

(b) $f(-3) = -f(3) = -(-4) = 4$

(d) Yes. Since $f(-2) = -f(2) = -(-1) = 1$ and $f(1) = -f(-1) = -2$, the Mean Value says that there exists at least one value c in $(-2, 1)$ such that

$$f'(c) = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{-2 - 1}{1 + 2} = -1.$$

(e) No, $\lim_{x \to 0} f(x)$ exists because f is continuous at $(0, 0)$.

(f) Yes, f is differentiable at $x = 2$.

8. No; the function is discontinuous at $x = 0$ which is in the interval $[-2, 1]$.

16. $g(x) = (x + 1)^3$

$g'(x) = 3(x + 1)^2$

Critical number: $x = -1$

Interval	$-\infty < x < -1$	$-1 < x < \infty$
Sign of $g'(x)$	$g'(x) > 0$	$g'(x) > 0$
Conclusion	Increasing	Increasing

18. $f(x) = \sin x + \cos x, \quad 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$

Critical numbers: $x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

Interval	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < 2\pi$
Sign of $f'(x)$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
Conclusion	Increasing	Decreasing	Increasing

24. $f(x) = (x+2)^2(x-4) = x^3 - 12x - 16$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x = 0$ when $x = 0$.

Point of inflection: $(0, -16)$

Test Interval	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f''(x)$	$f''(x) < 0$	$f''(x) > 0$
Conclusion	Concave downward	Concave upward

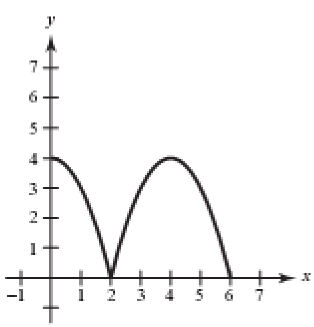
26. $h(t) = t - 4\sqrt{t+1}$ Domain: $[-1, \infty)$

$$h'(t) = 1 - \frac{2}{\sqrt{t+1}} = 0 \Rightarrow t = 3$$

$$h''(t) = \frac{1}{(t+1)^{3/2}}$$

$h''(3) = \frac{1}{8} > 0$ $(3, -5)$ is a relative minimum.

28.



36. $\lim_{x \to -\infty} \frac{\sqrt{x^2 + x}}{-2x} = 1/2$

38. $\lim_{x \to \infty} \frac{3x}{\sqrt{x^2+4}} = \lim_{x \to \infty} \frac{3}{\sqrt{1 + 4/x^2}} = 3$

42. $g(x) = \dfrac{5x^2}{x^2 + 2}$

$$\lim_{x \to \infty} \frac{5x^2}{x^2 + 2} = \lim_{x \to \infty} \frac{5}{1 + (2/x^2)} = 5$$

Horizontal asymptote: $y = 5$

52. $f(x) = (x^2 - 4)^2$

Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

$f'(x) = 4x(x^2 - 4) = 0$ when $x = 0, \pm 2$.

$f''(x) = 4(3x^2 - 4) = 0$ when $x = \pm\dfrac{2\sqrt{3}}{3}$.

$f''(0) < 0$

Therefore, $(0, 16)$ is a relative maximum.

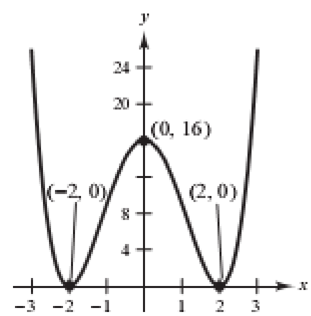
$f''(\pm 2) > 0$

Therefore, $(\pm 2, 0)$ are relative minima.

Points of inflection: $\left(\pm 2\sqrt{3}/3, 64/9\right)$

Intercepts: $(-2, 0)$, $(0, 16)$, $(2, 0)$

Symmetry with respect to y-axis



58. $f(x) = \dfrac{2x}{1 + x^2}$

Domain: $(-\infty, \infty)$; Range: $[-1, 1]$

$f'(x) = \dfrac{2(1 - x)(1 + x)}{(1 + x^2)^2} = 0$ when $x = \pm 1$.

$f''(x) = \dfrac{-4x(3 - x^2)}{(1 + x^2)^3} = 0$ when $x = 0, \pm\sqrt{3}$.

$f''(1) < 0$

Therefore, $(1, 1)$ is a relative maximum.

$f''(-1) > 0$

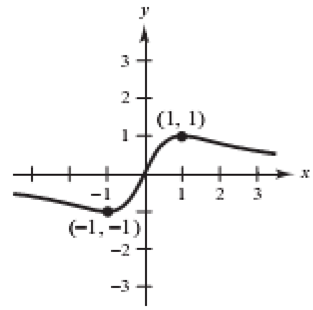
Therefore, $(-1, -1)$ is a relative minimum.

Points of inflection: $\left(-\sqrt{3}, -\sqrt{3}/2\right)$, $(0, 0)$, $\left(\sqrt{3}, \sqrt{3}/2\right)$

Intercept: $(0, 0)$

Symmetric with respect to the origin

Horizontal asymptote: $y = 0$

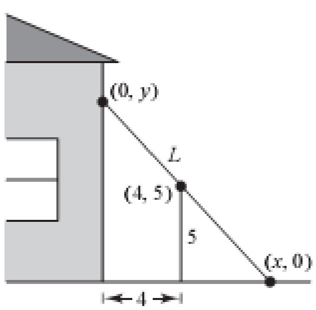


72. We have points $(0, y)$, $(x, 0)$, and $(4, 5)$. Thus,

$$m = \frac{y - 5}{0 - 4} = \frac{5 - 0}{4 - x} \text{ or } y = \frac{5x}{x - 4}.$$

Let $f(x) = L^2 = x^2 + \left(\dfrac{5x}{x - 4}\right)^2$

$$f'(x) = 2x + 50\left(\frac{x}{x - 4}\right)\left[\frac{x - 4 - x}{(x - 4)^2}\right] = 0$$



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$$x - \frac{100x}{(x-4)^3} = 0$$

$x[(x-4)^3 - 100] = 0$ when $x = 0$ or $x = 4 + \sqrt[3]{100}$.

$$L = \sqrt{x^2 + \frac{25x^2}{(x-4)^2}} = \frac{x}{x-4}\sqrt{(x-4)^2 + 25} = \frac{\sqrt[3]{100} + 4}{\sqrt[3]{100}}\sqrt{100^{2/3} + 25} \approx 12.7 \text{ feet}$$

87. $S = 4\pi r^2 \; dr = \Delta r = \pm 0.025$

$$dS = 8\pi r \; dr = 8\pi(9)(\pm 0.025)$$

$$= \pm 1.8\pi \text{ square cm}$$

$$\frac{dS}{S}(100) = \frac{8\pi r \; dr}{4\pi r^2}(100) = \frac{2 \; dr}{r}(100)$$

$$= \frac{2(\pm 0.025)}{9}(100) \approx \pm 0.56\%$$

$$V = \frac{4}{3}\pi r^3$$

$$dV = 4\pi r^2 \; dr = 4\pi(9)^2(\pm 0.025)$$

$$= \pm 8.1\pi \text{ cubic cm}$$

$$\frac{dV}{V}(100) = \frac{4\pi r^2 \; dr}{(4/3)\pi r^3}(100) = \frac{3 \; dr}{r}(100)$$

$$= \frac{3(\pm 0.025)}{9}(100) \approx \pm 0.83\%$$