

Answer Key for 11-16-07 Homework Assignment Page 267-269

[4.2] p.267-269 #1-41eoo, 47,51,55

$$1. \sum_{i=1}^5 (2i + 1) = 2 \sum_{i=1}^5 i + \sum_{i=1}^5 1 = 2(1 + 2 + 3 + 4 + 5) + 5 = 35$$

$$5. \sum_{k=1}^4 c = c + c + c + c = 4c$$

$$9. \sum_{j=1}^8 \left[5 \left(\frac{j}{8} \right) + 3 \right]$$

$$13. \frac{3}{n} \sum_{i=1}^n \left[2 \left(1 + \frac{3i}{n} \right)^2 \right]$$

$$17. \sum_{i=1}^{20} (i - 1)^2 = \sum_{i=1}^{19} i^2$$

$$= \left[\frac{19(20)(39)}{6} \right] = 2470$$

$$21. \text{sum seq}(x(\overline{\square}) 2 + 3, x, 1, 20, 1) = 2930 \quad (TI-82)$$

$$\sum_{i=1}^{20} (i^2 + 3) = \frac{20(20 + 1)(2(20) + 1)}{6} + 3(20)$$

$$= \frac{(20)(21)(41)}{6} + 60 = 2930$$

$$25. S = [3 + 3 + 5](1) = 11$$

$$s = [2 + 2 + 3](1) = 7$$

$$29. S(5) = 1 \left(\frac{1}{5} \right) + \frac{1}{6/5} \left(\frac{1}{5} \right) + \frac{1}{7/5} \left(\frac{1}{5} \right) + \frac{1}{8/5} \left(\frac{1}{5} \right) + \frac{1}{9/5} \left(\frac{1}{5} \right) = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} \approx 0.746$$

$$s(5) = \frac{1}{6/5} \left(\frac{1}{5} \right) + \frac{1}{7/5} \left(\frac{1}{5} \right) + \frac{1}{8/5} \left(\frac{1}{5} \right) + \frac{1}{9/5} \left(\frac{1}{5} \right) + \frac{1}{2} \left(\frac{1}{5} \right) = \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} \approx 0.646$$

$$33. \lim_{n \rightarrow \infty} \left[\left(\frac{18}{n^2} \right) \frac{n(n+1)}{2} \right] = \frac{18}{2} \lim_{n \rightarrow \infty} \left[\frac{n^2 + n}{n^2} \right] = \frac{18}{2}(1) = 9$$

$$37. \sum_{k=1}^n \frac{6k(k-1)}{n^3} = \frac{6}{n^3} \sum_{k=1}^n (k^2 - k) = \frac{6}{n^3} \left[\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} \right]$$

$$= \frac{6}{n^3} \left[\frac{2n^2 + 3n + 1 - 3n - 3}{6} \right] = \frac{1}{n^2} [2n^2 - 2] = 2 - \frac{2}{n^2} = S(n)$$

$$S(10) = 1.98$$

$$S(100) = 1.9998$$

$$S(1000) = 1.999998$$

$$S(10,000) = 1.99999998$$

$$41. \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n^3} (i-1)^2 = \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{i=1}^{n-1} i^2 = \lim_{n \rightarrow \infty} \frac{1}{n^3} \left[\frac{(n-1)(n)(2n-1)}{6} \right]$$

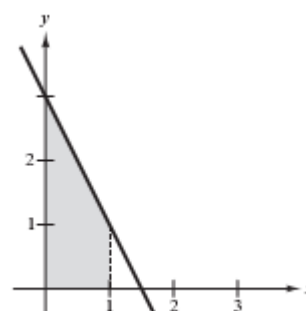
$$= \lim_{n \rightarrow \infty} \frac{1}{6} \left[\frac{2n^3 - 3n^2 + n}{n^3} \right] = \lim_{n \rightarrow \infty} \left[\frac{1}{6} \left(\frac{2 - (3/n) + (1/n^2)}{1} \right) \right] = \frac{1}{3}$$

$$47. y = -2x + 3 \text{ on } [0, 1]. \quad \left(\text{Note: } \Delta x = \frac{1-0}{n} = \frac{1}{n} \right)$$

$$s(n) = \sum_{i=1}^n f\left(\frac{i}{n}\right) \left(\frac{1}{n}\right) = \sum_{i=1}^n \left[-2\left(\frac{i}{n}\right) + 3 \right] \left(\frac{1}{n}\right)$$

$$= 3 - \frac{2}{n^2} \sum_{i=1}^n i = 3 - \frac{2(n+1)n}{2n^2} = 2 - \frac{1}{n}$$

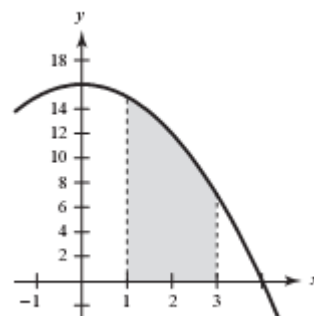
$$\text{Area} = \lim_{n \rightarrow \infty} s(n) = 2$$



51. $y = 16 - x^2$ on $[1, 3]$. (Note: $\Delta x = \frac{2}{n}$)

$$\begin{aligned} s(n) &= \sum_{i=1}^n f\left(1 + \frac{2i}{n}\right)\left(\frac{2}{n}\right) = \sum_{i=1}^n \left[16 - \left(1 + \frac{2i}{n}\right)^2\right]\left(\frac{2}{n}\right) \\ &= \frac{2}{n} \sum_{i=1}^n \left[15 - \frac{4i^2}{n^2} - \frac{4i}{n}\right] \\ &= \frac{2}{n} \left[15n - \frac{4}{n^2} \frac{n(n+1)(2n+1)}{6} - \frac{4}{n} \frac{n(n+1)}{2}\right] \\ &= 30 - \frac{8}{6n^2}(n+1)(2n+1) - \frac{4}{n}(n+1) \end{aligned}$$

$$\text{Area} = \lim_{n \rightarrow \infty} s(n) = 30 - \frac{8}{3} - 4 = \frac{70}{3} = 23\frac{1}{3}$$



55. $y = x^2 - x^3$ on $[-1, 1]$. (Note: $\Delta x = \frac{1 - (-1)}{n} = \frac{2}{n}$)

Again, $T(n)$ is neither an upper nor a lower sum.

$$\begin{aligned} T(n) &= \sum_{i=1}^n f\left(-1 + \frac{2i}{n}\right)\left(\frac{2}{n}\right) = \sum_{i=1}^n \left[\left(-1 + \frac{2i}{n}\right)^2 - \left(-1 + \frac{2i}{n}\right)^3 \right] \left(\frac{2}{n}\right) \\ &= \sum_{i=1}^n \left[\left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right) - \left(-1 + \frac{6i}{n} - \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right) \right] \left(\frac{2}{n}\right) \\ &= \sum_{i=1}^n \left[2 - \frac{10i}{n} + \frac{16i^2}{n^2} - \frac{8i^3}{n^3} \right] \left(\frac{2}{n}\right) = \frac{4}{n} \sum_{i=1}^n 1 - \frac{20}{n^2} \sum_{i=1}^n i + \frac{32}{n^3} \sum_{i=1}^n i^2 - \frac{16}{n^4} \sum_{i=1}^n i^3 \\ &= \frac{4}{n}(n) - \frac{20}{n^2} \cdot \frac{n(n+1)}{2} + \frac{32}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{16}{n^4} \cdot \frac{n^2(n+1)^2}{4} \\ &= 4 - 10\left(1 + \frac{1}{n}\right) + \frac{16}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 4\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) \end{aligned}$$

$$\text{Area} = \lim_{n \rightarrow \infty} T(n) = 4 - 10 + \frac{32}{3} - 4 = \frac{2}{3}$$

