

## 6.2 Differential Equations: Growth and Decay

Answer Key for Examples of Today's Homework Pages 418-419

$$4. \quad \frac{dy}{dx} = 4 - y$$

$$\frac{dy}{4 - y} = dx$$

$$\int \frac{-1}{4 - y} dy = \int -dx$$

$$\ln|4 - y| dy = -x + C_1$$

$$4 - y = e^{-x+C_1} = Ce^{-x}$$

$$y = 4 - Ce^{-x}$$

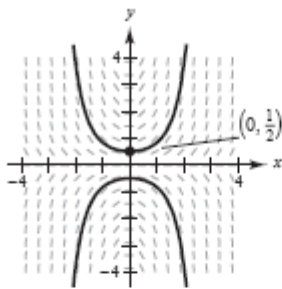
$$12. \quad \frac{dP}{dt} = k(10 - t)$$

$$\int \frac{dP}{dt} dt = \int k(10 - t) dt$$

$$\int dP = -\frac{k}{2}(10 - t)^2 + C$$

$$P = -\frac{k}{2}(10 - t)^2 + C$$

16. (a)



$$(b) \quad \frac{dy}{dx} = xy, \quad \left(0, \frac{1}{2}\right)$$

$$\frac{dy}{y} = x dx$$

$$\ln|y| = \frac{x^2}{2} + C$$

$$y = e^{x^2/2+C} = C_1 e^{x^2/2}$$

$$\left(0, \frac{1}{2}\right): \frac{1}{2} = C_1 e^0 \Rightarrow C_1 = \frac{1}{2} \Rightarrow y = \frac{1}{2} e^{x^2/2}$$

## 6.2 Differential Equations: Growth and Decay

20.  $\frac{dy}{dt} = \frac{3}{4}y, (0, 10)$

$$\int \frac{dy}{y} = \int \frac{3}{4} dt$$

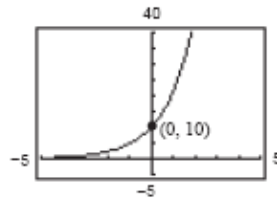
$$\ln y = \frac{3}{4}t + C_1$$

$$y = e^{(3/4)t + C_1}$$

$$= e^{C_1} e^{(3/4)t} = Ce^{3t/4}$$

$$10 = Ce^0 \Rightarrow C = 10$$

$$y = 10e^{3t/4}$$



22.  $\frac{dN}{dt} = kN$

$$N = Ce^{kt} \quad (\text{Theorem 5.16})$$

$$(0, 250): C = 250$$

$$(1, 400): 400 = 250e^k \Rightarrow k = \ln \frac{400}{250} = \ln \frac{8}{5}$$

$$\begin{aligned} \text{When } t = 4, N &= 250e^{4 \ln(8/5)} = 250e^{\ln(8/5)^4} \\ &= 250\left(\frac{8}{5}\right)^4 = \frac{8192}{5}. \end{aligned}$$

26.  $y = Ce^{kt}, (0, 4), \left(5, \frac{1}{2}\right)$

$$C = 4$$

$$y = 4e^{kt}$$

$$\frac{1}{2} = 4e^{5k}$$

$$k = \frac{\ln(1/8)}{5} \approx -0.4159$$

$$y = 4e^{-0.4159t}$$

34. Since the half-life is 1599 years,

$$\frac{1}{2} = 1e^{k(1599)}$$

$$k = \frac{1}{1599} \ln\left(\frac{1}{2}\right).$$

Since there are 1.5 g after 1000 years,

$$1.5 = Ce^{[\ln(1/2)/1599](1000)}$$

$$C \approx 2.314.$$

Hence, the initial quantity is approximately 2.314 g.

$$\text{When } t = 10,000, y = 2.314e^{[\ln(1/2)/1599](10,000)}$$

$$\approx 0.03 \text{ g.}$$

## 6.2 Differential Equations: Growth and Decay

39. Since the half-life is 24,100 years,

$$\frac{1}{2} = 1e^{k(24,100)}$$

$$k = \frac{1}{24,100} \ln\left(\frac{1}{2}\right).$$

Since there are 2.1 grams after 1000 years,

$$2.1 = Ce^{[\ln(1/2)/24,100](1000)}$$

$$C \approx 2.161.$$

Thus, the initial quantity is approximately 2.161 g.

$$\begin{aligned} \text{When } t = 10,000, y &= 2.161e^{[\ln(1/2)/24,100](10,000)} \\ &\approx 1.62 \text{ g.} \end{aligned}$$

## 6.2 Differential Equations: Growth and Decay

Answer Key for Today's Homework Assignment Pages 418-419

[6.2] p.418-419 #1,5,9,11,15,21,25,29,33,37,42

1.  $\frac{dy}{dx} = x + 2$

$$y = \int (x + 2) dx = \frac{x^2}{2} + 2x + C$$

5.  $y' = \frac{5x}{y}$

$$yy' = 5x$$

$$\int yy' dx = \int 5x dx$$

$$\int y dy = \int 5x dx$$

$$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C_1$$

$$y^2 - 5x^2 = C$$

9.  $(1 + x^2)y' - 2xy = 0$

$$y' = \frac{2xy}{1 + x^2}$$

$$\frac{y'}{y} = \frac{2x}{1 + x^2}$$

$$\int \frac{y'}{y} dx = \int \frac{2x}{1 + x^2} dx$$

$$\int \frac{dy}{y} = \int \frac{2x}{1 + x^2} dx$$

$$\ln|y| = \ln(1 + x^2) + C_1$$

$$\ln|y| = \ln(1 + x^2) + \ln C$$

$$\ln|y| = \ln[C(1 + x^2)]$$

$$y = C(1 + x^2)$$

11.  $\frac{dQ}{dt} = \frac{k}{t^2}$

$$\int \frac{dQ}{dt} dt = \int \frac{k}{t^2} dt$$

$$\int dQ = -\frac{k}{t} + C$$

$$Q = -\frac{k}{t} + C$$

6.2 Differential Equations: Growth and Decay

(b)  $\frac{dy}{dx} = x(6 - y), (0, 0)$

$$\frac{dy}{y - 6} = -x dx$$

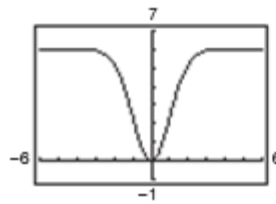
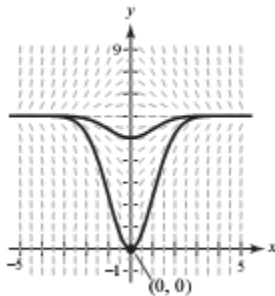
$$\ln|y - 6| = \frac{-x^2}{2} + C$$

$$y - 6 = e^{-x^2/2+C} = C_1 e^{-x^2/2}$$

$$y = 6 + C_1 e^{-x^2/2}$$

$(0, 0): 0 = 6 + C_1 \Rightarrow C_1 = -6 \Rightarrow y = 6 - 6e^{-x^2/2}$

15. (a)



21.  $\frac{dy}{dx} = ky$

$$y = Ce^{kx} \quad (\text{Theorem 5.16})$$

$(0, 4): 4 = Ce^0 = C$

$(3, 10): 10 = 4e^{3k} \Rightarrow k = \frac{1}{3} \ln\left(\frac{5}{2}\right)$

When  $x = 6, y = 4e^{1/3 \ln(5/2)(6)} = 4e^{\ln(5/2)^2}$

$$= 4\left(\frac{5}{2}\right)^2 = 25.$$

25.  $y = Ce^{kt}, \left(0, \frac{1}{2}\right), (5, 5)$

$$C = \frac{1}{2}$$

$$y = \frac{1}{2}e^{kt}$$

$$5 = \frac{1}{2}e^{5k}$$

$$k = \frac{\ln 10}{5}$$

$$y = \frac{1}{2}e^{(\ln 10/5)t} = \frac{1}{2}(10^{t/5}) \text{ or } y \approx \frac{1}{2}e^{0.4605t}$$

29. In the model  $y = Ce^{kt}$ ,  $C$  represents the initial value of  $y$  (when  $t = 0$ ).  $k$  is the proportionality constant.

## 6.2 Differential Equations: Growth and Decay

33. Since the initial quantity is 10 grams,

$$y = 10e^{kt}.$$

Since the half-life is 1599 years,

$$5 = 10e^{k(1599)}$$

$$k = \frac{1}{1599} \ln\left(\frac{1}{2}\right).$$

Thus,  $y = 10e^{[\ln(1/2)/1599]t}$ .

When  $t = 1000$ ,  $y = 10e^{[\ln(1/2)/1599](1000)} \approx 6.48$  g.

When  $t = 10,000$ ,  $y \approx 0.13$  g.

37. Since the initial quantity is 5 grams,  $C = 5$ .

Since the half-life is 5715 years,

$$2.5 = 5e^{k(5715)}$$

$$k = \frac{1}{5715} \ln\left(\frac{1}{2}\right).$$

When  $t = 1000$  years,  $y = 5e^{[\ln(1/2)/5715](1000)}$   
 $\approx 4.43$  g.

When  $t = 10,000$  years,  $y = 5e^{[\ln(1/2)/5715](10,000)}$   
 $\approx 1.49$  g.

42.  $y = Ce^{kt}$

$$\frac{1}{2}C = Ce^{k(5715)}$$

$$k = \frac{1}{5715} \ln\left(\frac{1}{2}\right)$$

$$0.15C = Ce^{[\ln(1/2)/5715]t}$$

$$\ln(0.15) = \frac{\ln\left(\frac{1}{2}\right)t}{5715}$$

$$t \approx 15,641.8 \text{ years}$$