

Answer Key for Today's Homework Assignment Pages 409-410

[6.1] p.409-410 #1-35 odd

1. Differential equation: $y' = 4y$

Solution: $y = Ce^{4x}$

Check: $y' = 4Ce^{4x} = 4y$

3. Differential equation: $y' = \frac{2xy}{x^2 - y^2}$

Solution: $x^2 + y^2 = Cy$

Check: $2x + 2yy' = Cy'$

$$\begin{aligned} y' &= \frac{-2x}{(2y - C)} \\ y' &= \frac{-2xy}{2y^2 - Cy} \\ &= \frac{-2xy}{2y^2 - (x^2 + y^2)} \\ &= \frac{-2xy}{y^2 - x^2} \\ &= \frac{2xy}{x^2 - y^2} \end{aligned}$$

5. Differential equation: $y'' + y = 0$

Solution: $y = C_1 \cos x + C_2 \sin x$

Check: $y' = -C_1 \sin x + C_2 \cos x$

$y'' = -C_1 \cos x - C_2 \sin x$

$y'' + y = -C_1 \cos x - C_2 \sin x + C_1 \cos x + C_2 \sin x = 0$

7. Differential Equation: $y'' + y = \tan x$

$$y = -\cos x \ln|\sec x + \tan x|$$

$$\begin{aligned} y' &= (-\cos x) \frac{1}{\sec x + \tan x} (\sec x \cdot \tan x + \sec^2 x) + \sin x \ln|\sec x + \tan x| \\ &= \frac{(-\cos x)}{\sec x + \tan x} (\sec x)(\tan x + \sec x) + \sin x \ln|\sec x + \tan x| \\ &= -1 + \sin x \ln|\sec x + \tan x| \end{aligned}$$

$$\begin{aligned} y'' &= (\sin x) \frac{1}{\sec x + \tan x} (\sec x \cdot \tan x + \sec^2 x) + \cos x \ln|\sec x + \tan x| \\ &= (\sin x)(\sec x) + \cos x \ln|\sec x + \tan x| \end{aligned}$$

Substituting,

$$\begin{aligned} y'' + y &= (\sin x)(\sec x) + \cos x \ln|\sec x + \tan x| - \cos x \ln|\sec x + \tan x| \\ &= \tan x. \end{aligned}$$

9. $y = \sin x \cos x - \cos^2 x$

$$\begin{aligned} y' &= -\sin^2 x + \cos^2 x + 2 \cos x \sin x \\ &= -1 + 2 \cos^2 x + \sin 2x \end{aligned}$$

Differential Equation:

$$\begin{aligned} 2y + y' &= 2(\sin x \cos x - \cos^2 x) + (-1 + 2 \cos^2 x + \sin 2x) \\ &= 2 \sin x \cos x - 1 + \sin 2x \\ &= 2 \sin 2x - 1 \end{aligned}$$

Initial condition:

$$y\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{4} - \cos^2 \frac{\pi}{4} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2}\right)^2 = 0$$

11. $y = 6e^{-2x^2}$

$$y' = 6e^{-2x^2}(-4x) = -24xe^{-2x^2}$$

Differential Equation:

$$y' = -24xe^{-2x^2} = -4x(6e^{-2x^2}) = -4xy$$

Initial condition:

$$y(0) = 6e^{-2(0)^2} = 6e^0 = 6(1) = 6$$

In Exercises 13–18, the differential equation is $y^{(4)} - 16y = 0$.

13. $y = 3 \cos x$

$$y^{(4)} = 3 \cos x$$

$$y^{(4)} - 16y = -45 \cos x \neq 0,$$

No

15. $y = e^{-2x}$

$$y^{(4)} = 16e^{-2x}$$

$$y^{(4)} - 16y = 16e^{-2x} - 16e^{-2x} = 0,$$

Yes

17. $y = C_1 e^{2x} + C_2 e^{-2x} + C_3 \sin 2x + C_4 \cos 2x$

$$y^{(4)} = 16C_1 e^{2x} + 16C_2 e^{-2x} + 16C_3 \sin 2x + 16C_4 \cos 2x$$

$$y^{(4)} - 16y = 0,$$

Yes

In 19–24, the differential equation is $xy' - 2y = x^3 e^x$.

19. $y = x^2, y' = 2x$

$$xy' - 2y = x(2x) - 2(x^2) = 0 \neq x^3 e^x,$$

No

21. $y = x^2(2 + e^x), y' = x^2(e^x) + 2x(2 + e^x)$

$$xy' - 2y = x[x^2 e^x + 2xe^x + 4x] - 2[x^2 e^x + 2x^2] = x^3 e^x,$$

Yes

23. $y = \ln x, y' = \frac{1}{x}$

$$xy' - 2y = x\left(\frac{1}{x}\right) - 2 \ln x \neq x^3 e^x,$$

No

25. $y = Ce^{-x/2}$ passes through $(0, 3)$.

$$3 = Ce^0 = C \implies C = 3$$

Particular solution: $y = 3e^{-x/2}$

27. $y^2 = Cx^3$ passes through $(4, 4)$.

$$16 = C(64) \implies C = \frac{1}{4}$$

Particular solution: $y^2 = \frac{1}{4}x^3$ or $4y^2 = x^3$

29. Differential equation: $4yy' - x = 0$

General solution: $4y^2 - x^2 = C$

Particular solutions: $C = 0$, Two intersecting lines
 $C = \pm 1$, $C = \pm 4$, Hyperbolas

31. Differential equation: $y' + 2y = 0$

General solution: $y = Ce^{-2x}$

$$y' + 2y = C(-2)e^{-2x} + 2(Ce^{-2x}) = 0$$

Initial condition: $y(0) = 3, 3 = Ce^0 = C$

Particular solution: $y = 3e^{-2x}$

33. Differential equation: $y'' + 9y = 0$

General solution: $y = C_1 \sin 3x + C_2 \cos 3x$

$$y' = 3C_1 \cos 3x - 3C_2 \sin 3x,$$

$$y'' = -9C_1 \sin 3x - 9C_2 \cos 3x$$

$$y'' + 9y = (-9C_1 \sin 3x - 9C_2 \cos 3x) + 9(C_1 \sin 3x + C_2 \cos 3x) = 0$$

35. Differential equation: $x^2y'' - 3xy' + 3y = 0$

General solution: $y = C_1x + C_2x^3$

$$y' = C_1 + 3C_2x^2, y'' = 6C_2x$$

$$x^2y'' - 3xy' + 3y = x^2(6C_2x) - 3x(C_1 + 3C_2x^2) + 3(C_1x + C_2x^3) = 0$$

Initial conditions: $y(2) = 0, y'(2) = 4$

$$0 = 2C_1 + 8C_2$$

$$y' = C_1 + 3C_2x^2$$

$$4 = C_1 + 12C_2$$

$$\left. \begin{array}{l} C_1 + 4C_2 = 0 \\ C_1 + 12C_2 = 4 \end{array} \right\} C_2 = \frac{1}{2}, C_1 = -2$$

Particular solution: $y = -2x + \frac{1}{2}x^3$