Answer Key for WebAssign Day 92 Homework Assignment

5.2 Growth and Decay

WA Larson
#1-8 [5.2] p.382-383 #4,5,10,11,13,19,21,29

4.

$$\frac{dy}{dx} = 6 - y$$
$$\frac{dy}{6 - y} = dx$$
$$\int \frac{-1}{6 - y}\, dy = \int -dx$$
$$\ln|6 - y|dy = -x + C_1$$
$$6 - y = e^{-x+C_1} = Ce^{-x}$$
$$y = 6 - Ce^{-x}$$

5.

$$y' = \frac{5x}{y}$$
$$yy' = 5x$$
$$\int yy'\, dx = \int 5x\, dx$$
$$\int y\, dy = \int 5x\, dx$$
$$\frac{1}{2}y^2 = \frac{5}{2}x^2 + C_1$$
$$y^2 - 5x^2 = C$$

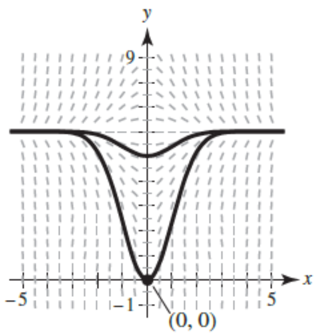
10.

$$xy + y' = 100x$$
$$y' = 100x + xy = x(100 - y)$$
$$\frac{y'}{100 - y} = x$$
$$\int \frac{y'}{100 - y}\, dx = \int x\, dx$$
$$\int \frac{1}{100 - y}\, dy = \int x\, dx$$
$$-\ln(100 - y) = \frac{x^2}{2} + C_1$$
$$\ln(100 - y) = -\frac{x^2}{2} - C_1$$
$$100 - y = e^{-(x^2/2) - C_1}$$
$$-y = e^{-C_1}e^{-x^2/2} - 100$$
$$y = 100 - Ce^{-x^2/2}$$

11.

$$\frac{dQ}{dt} = \frac{k}{t^2}$$
$$\int \frac{dQ}{dt}\, dt = \int \frac{k}{t^2}\, dt$$
$$\int dQ = -\frac{k}{t} + C$$
$$Q = -\frac{k}{t} + C$$

13. (a)



(b) $$\frac{dy}{dx} = x(6 - y), \quad (0, 0)$$

$$\frac{dy}{y - 6} = -x\,dx$$

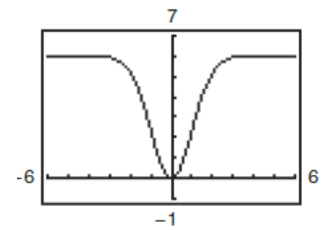
$$\ln|y - 6| = \frac{-x^2}{2} + C$$

$$y - 6 = e^{-x^2/2 + C} = C_1 e^{-x^2/2}$$

$$y = 6 + C_1 e^{-x^2/2}$$

$$(0, 0):\ 0 = 6 + C_1 \Rightarrow C_1 = -6$$

$$y = 6 - 6e^{-x^2/2}$$



19. $$\frac{dN}{dt} = kN$$

$N = Ce^{kt}$ (Theorem 5.1)

$(0, 250)$: $C = 250$

$(1, 400)$: $400 = 250e^{k} \Rightarrow k = \ln\frac{400}{250} = \ln\frac{8}{5}$

$N = 250e^{\ln(8/5)t} \approx 250e^{0.4700t}$

When $t = 4$, $N = 250e^{4\ln(8/5)} = 250e^{\ln(8/5)^4}$

$$= 250\left(\frac{8}{5}\right)^4 = \frac{8192}{5}.$$

21. $y = Ce^{kt}$, $\left(0, \frac{1}{2}\right)$, $(5, 5)$

$$C = \frac{1}{2}$$

$$y = \frac{1}{2}e^{kt}$$

$$5 = \frac{1}{2}e^{5k}$$

$$k = \frac{\ln 10}{5}$$

$$y = \frac{1}{2}e^{[(\ln 10)/5]t} = \frac{1}{2}\left(10^{t/5}\right) \text{ or } y \approx \frac{1}{2}e^{0.4605t}$$

29. Because the initial quantity is 20 grams,

$y = 20e^{kt}$.

Because the half-life is 1599 years,

$10 = 20e^{k(1599)}$

$k = \frac{1}{1599}\ln\left(\frac{1}{2}\right)$.

So, $y = 20e^{\left[\ln(1/2)/1599\right]t}$.

When $t = 1000$, $y = 20e^{\left[\ln(1/2)/1599\right](1000)} \approx 12.96$ g.

When $t = 10{,}000$, $y \approx 0.26$ g.